Light scattering from a periodically modulated two dimensional electron gas with partially filled Landau levels

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Abstract

We study light scattering from a periodically modulated two dimensional electron gas in a perpendicular magnetic field. If a subband is partially filled, the imaginary part of the dielectric function as a function of frequency contains additional discontinuities to the case of completely filled subbands. The positions of the discontinuities may be determined from the partial filling factor and the height of the discontinuity can be directly related to the modulation potential. The light scattering cross section contains a new peak which is absent for integer filling.

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Since the Weiss oscillation was first observed in the magneto-resistivity several years ago,[1] a considerable amount of work, both experimentally and theoretically, have been carried out on the electronic and transport properties of two dimensional electron system under a periodic potential and a constant magnetic field.[2–10] Most recent works include the DC transport in a strong anti-dot system[11,12] and the observation of quantum fractal like energy spectrum.[13] Weiss oscillation can be understood as a type of commensurability oscillation originated from the interplay of two different length scales of the system, the periodicity of the modulation potential a, and the radius of the cyclotron motion R_c. The DC resistivity has a set of minima whenever the condition $2R_c = (n - 1/4)a$ is satisfied, where n is any integer. Most of the current investigations have been limited to the case where the external field has zero frequency. In a recent paper by Stewart and Zhang,[14] the dielectric response at finite frequency and wave-numbers was calculated. Their result indicates that the modulation induced structure at finite frequency is much richer than that of the static case. Most noticeably, the electron-hole pair excitation contains a set of sub-singularities at the excitation band edges.

In this brief report, we investigate the density response function within the random-phase-approximation for a two dimensional electron gas under a weak periodic modulation and a constant magnetic field. We shall pay special attention to the case where the Fermi level lies within a Landau band. In what follows, we shall show that this partially filled Landau band at the Fermi level has profound effect on the dielectric response and in turn alters the light scattering cross section from such a system. Our main results are: (i) Electron-hole pair excitation has a step-like (discontinuous) behavior around $\omega = n\omega_c$. This step can be determined analytically for the case of weak modulation. Thus the amplitude of the modulation potential, which so far has been rather difficult to measure experimentally, can now be precisely determined. (ii) Also around $\omega = n\omega_c$, the real part of the dielectric function has a logarithm divergence. (iii) When the new effects of (i) and (ii) are included in the density response function, a new sharp peak can be observed in the light scattering cross section.

We consider a two dimensional electron gas where a static magnetic field B is perpendicular to the plane. A weak periodic potential is applied in the x-direction

$$V(x) = V_0 \cos(Kx)\,, \tag{1}$$

where $K = 2\pi/a$ and a is the period of the modulation. In the Landau gauge the single-particle wave functions are of the form $\psi_{nk}(x,y) \propto \exp(\imath k_y y)\phi_{nx_0}(x)$ where $x_0 = k_y l^2$ (magnetic length, $l = [\hbar/(m^*\omega_c)]^{1/2}$) is the center coordinate and $\phi_{nx_0}(x)$ is the eigenfunction of the one-dimensional Hamiltonian

$$H = -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + \frac{1}{2}m^*\omega_c^2(x-x_0)^2 + V_0\cos(Kx)\,, \tag{2}$$

where m^* is the effective mass and $\omega_c = eB/(m^*c)$ is the cyclotron frequency. We assume that the modulation potential is a weak perturbation, $V_0 \ll E_F$, where E_F is the Fermi energy. The energy spectrum to linear order in the modulation potential is[3]

$$E_n(x_0) = \Big(n + \frac{1}{2}\Big)\hbar\omega_c + U_n\cos(Kx_0)\,, \tag{3}$$

where $U_n = V_0\exp(-\mathcal{H}/2)L_n(\mathcal{H})$, $\mathcal{H} = (Kl)^2/2$ and $L_n(\mathcal{H})$ is a Laguerre polynomial.

In order to simplify the discussion of the discontinuity in the imaginary part of the dielectric function as a function of the frequency let us consider only vertical transition. The energy difference for the transition from state m', x_0 to state $m'+m, x_0$ is then ($m > 0$)

$$E_{m+m'}(x_0) - E_{m'}(x_0) = m\hbar\omega_c + (U_{m+m'} - U_{m'})\cos(Kx_0). \tag{4}$$

For sufficiently small $\mathcal{H}$ we may approximate $L_n(\mathcal{H}) = 1 - n\mathcal{H}$, so that $U_{m+m'} - U_m = -V_0 m\mathcal{H}\exp(-\mathcal{H}/2)$. We also have $U_n(\mathcal{H}) > 0$ for small n and $\mathcal{H}$. This means that the energy difference between the states $m+m', x_0$ and m', x_0 is less than $m\hbar\omega_c$ for $0 < Kx_0 < \pi/2$ and $3\pi/2 < Kx_0 < 2\pi$. On the other hand the energy difference is larger than $m\hbar\omega_c$ for $\pi/2 < Kx_0 < 3\pi/2$. Let us now consider a situation where the last Landau band (with index n_F) is half filled. At $\omega \sim \omega_c$, two transition processes ($n_F - 1 \to n_F$ and $n_F \to n_F + 1$)

contribute to the electron-hole pair excitation. However the transition $(n_F - 1 \to n_F)$ only contributes for $\omega < \omega_c$ while for $\omega > \omega_c$ only the transition $(n_F \to n_F + 1)$ contributes (see Fig. (1)). At $\omega = \omega_c$ the contributions from these two distinct transitions are different and thus a discontinuity occurs at $\omega = \omega_c$. One can immediately generalize this conclusion for excitations around $\omega \sim m\omega_c$ where a similar discontinuity occurs due to the different contributions from the $(n_F - m \to n_F)$ transition and the $(n_F \to n_F + m)$ transition. Below we shall derive an analytical expression for this discontinuity for strictly vertical transition.

The case of half-filling is a quite special situation but actually exhibits the general discontinuous behavior of the pair excitation in modulated systems. If the last subband is less than half-filled, the discontinuity in the imaginary part of the dielectric function will appear for frequencies larger than $m\omega_c$. If the last sub-band is more than half-filled, the discontinuity in the imaginary part of the dielectric function will appear for frequencies lower than $m\omega_c$. In this way the imaginary part of the dielectric function also provides direct information about the filling of the last subband. For integer fillings, such discontinuity will disappear. For non-integer fillings but non-vertical excitations ($q_y \neq 0$), such discontinuity will still be present.

The dielectric function within the random-phase approximation is

$$\epsilon(q_x, \omega) = 1 - 2\pi r_s \frac{k_F}{q_x} \hbar\omega_c \times \sum_{n \neq n'} \sum_{x_0} C_{nn'} \frac{f_{n,x_0} - f_{n',x_0}}{E_{n,x_0} - E_{n',x_0} + \hbar(\omega + \imath\delta)} \,, \tag{5}$$

where r_s is the plasma parameters, f_{n,x_0} is the Fermi occupation function. The coefficient $C_{nn'}$ $(n > n')$ is given as

$$C_{nn'} = \frac{n'!}{n!} X^{n-n'} \left[L_{n'}^{n-n'}(X) \right]^2 \exp(-X) \tag{6}$$

with $X = (q_x l)^2/2$ and $L_n^m(X)$ is an associated Laguerre polynomial.

For positive frequencies the imaginary part of the dielectric function is[14]

$$\mathrm{Im}[\epsilon(q, \omega)] = -2\pi^2 r_s (k_F/q) \hbar\omega_c \times \sum_{m=1}^{\infty} \sum_{m'=0}^{n_F} C_{m+m',m'} (f_{m+m',x_j} - f_{m',x_j}) Q^{mm'} \,, \tag{7}$$

where the function

$$Q^{mm'} = \frac{\theta(|U_{m+m'} - U_{m'} - |m\hbar\omega_c - \hbar\omega|)}{\sqrt{(U_{m+m'} - U_{m'})^2 - (m\hbar\omega_c - \hbar\omega)^2}} \tag{8}$$

gives the square-root singularities. In (7) we also sum over the simple roots x_j given by

$$\cos Kx_j = -\frac{m\hbar\omega_c - \hbar\omega}{U_{m+m'} - U_{m'}}\,. \tag{9}$$

For small $(m\hbar\omega_c - \hbar\omega)/(U_{m+m'} - U_{m'})$ we see that in the interval $0 < Kx_0 < 2\pi$ the solutions of (9) are $Kx_0 = \pi/2 + (m\hbar\omega_c - \hbar\omega)/(U_{m+m'} - U_{m'})$ and $Kx_1 = 3\pi/2 - (m\hbar\omega_c - \hbar\omega)/(U_{m+m'} - U_{m'})$. We introduce a small energy shift $\delta = \omega - 4\omega_c$ and find from (7) and (9) that for $\delta = 0^-$

$$\begin{aligned} &\mathrm{Im}[\epsilon(q, \delta = 0^-)] = 4\pi^2 r_S (k_F/q)\hbar\omega_c \times \\ &\left[\frac{C_{5,1}}{|U_5 - U_1|} + \frac{C_{6,2}}{|U_6 - U_2|} + \frac{C_{7,3}}{|U_7 - U_3|} + \frac{C_{8,4}}{|U_8 - U_4|}\right]. \end{aligned} \tag{10}$$

On the contrary for $\delta = 0^+$ we find

$$\begin{aligned} &\mathrm{Im}[\epsilon(q, \delta = 0^+)] = 4\pi^2 r_s (k_F/q)\hbar\omega_c \times \\ &\left[\frac{C_{6,2}}{|U_6 - U_2|} + \frac{C_{7,3}}{|U_7 - U_3|} + \frac{C_{8,4}}{|U_8 - U_4|} + \frac{C_{9,5}}{|U_9 - U_5|}\right]. \end{aligned} \tag{11}$$

Therefore

$$\begin{aligned} \Delta\mathrm{Im}[\epsilon(q_x)] &= \mathrm{Im}[\epsilon(q, \delta = 0^+)] - \mathrm{Im}[\epsilon(q, \delta = 0^-)] \\ &= 4\pi^2 r_S (k_F/q)\hbar\omega_c \times \\ &\quad \left[\frac{C_{5,1}}{|U_5 - U_1|} - \frac{C_{9,5}}{|U_9 - U_5|}\right]. \end{aligned} \tag{12}$$

Similarly the discontinuity around $\omega = 3\omega_c$ is given as $C_{5,2}/|U_5 - U_2| - C_{8,5}/|U_8 - U_5|$, and etc. Since the first order potential element U_n is linearly proportional to the modulation potential, we see that apart from numerical constants the discontinuity in the imaginary part of the dielectric function is determined directly by the strength of the modulation potential.

Note also that the imaginary part of the dielectric function should contain i sub-singularities around each frequency band even for partially filled subands as long as $q_y = 0$.

For nonzero q_y, in principle one may have $i+1$ subsingularities as stated in Ref.[14] where i is an integer and counts the resonance frequency.

For our numerical evaluation of the dielectric function we have employed the following parameters for a typical modulated GaAs/Al_xGa_{1-x}As heterostructure, $\kappa = 13$, $r_s = 0.73$, $E_F = 10$ meV, and $m^* = 0.067m_e$. The amplitude and period of the modulation potential are V_0 =1 meV, and $a = 300$ nm, respectively. The calculation was done at zero temperature with $q_x = 0.2k_F$, $q_y = 0$. The numerical parameters are thus the same as used in Ref. [14] except that we have a strictly vertical transition $q_y = 0$ instead of the small $q_y = 1 \times 10^6$ m^{-1}.

We show in Fig. (2) the imaginary part of the dielectric function around $\omega = 4\omega_c$. The discontinuity around $\omega = 4\omega_c$ is clearly resolved together with the four subsingularities on each side of the frequency band.

If exchange and correlation effect are neglected, the imaginary part of the dielectric function is proportional to the cross section of spin-density excitations which may be measured in Raman scattering when the polarization of the incoming and scattered light are perpendicular.[15] Within this approximation the discontinuity step in the imaginary part of the dielectric function is therefore directly measurable.

The real part of the dielectric function is

$$\mathrm{Re}[\epsilon(q,\omega)] = 1 - 4\pi^2 r_s \frac{k_F}{q}\hbar\omega_c \times \sum_{m=1}^{\infty}\sum_{m'=0}^{n_F} C_{m+m',m'}\Big(I_1 - I_2 + I_3 - I_4\Big), \tag{13}$$

where

$$I_1 = I(m\hbar\omega_c + \hbar\omega, U_{m+m'}, E_F - E_{m+m'}, U_{m+m'}) \tag{14a}$$

$$I_2 = I(m\hbar\omega_c + \hbar\omega, U_{m'}, E_F - E_{m'}, U_{m'}) \tag{14b}$$

$$I_3 = I(m\hbar\omega_c - \hbar\omega, U_{m+m'}, E_F - E_{m+m'}, U_{m+m'}) \tag{14c}$$

$$I_4 = I(m\hbar\omega_c - \hbar\omega, U_{m'}, E_F - E_{m'}, U_{m'}) \tag{14d}$$

are given by the Cauchy principal value integral

$$I(a,b,c,d) \equiv \frac{1}{2\pi}\int_0^\infty d\phi\theta(c-d\cos\phi)P\frac{1}{a+b\cos\phi}\,. \tag{15}$$

The integral may be found by a complex contour integration. For $(c/d)^2 > 1$, which is the case for completely occupied or unoccupied sub-bands, the integral is simply

$$I = \begin{cases} \theta(c)2\pi K & (a/b)^2 > 1 \\ 0 & (a/b)^2 < 1 \end{cases}, \tag{16}$$

where the factor K is

$$K = \frac{1}{2\pi a\sqrt{|1-(b/a)^2|}}\,. \tag{17}$$

For the partially filled sub-bands where $(c/d)^2 < 1$ we find

$$I = K \times \begin{cases} 2\pi - 4\arctan\gamma & (a/b)^2 > 1, d > 0 \\ 4\arctan\gamma & (a/b)^2 > 1, d < 0 \\ -2\ln|\frac{1-\gamma}{1+\gamma}| & (a/b)^2 < 1, d > 0 \\ 2\ln|\frac{1-\gamma}{1+\gamma}| & (a/b)^2 < 1, d < 0 \end{cases}, \tag{18}$$

where

$$\gamma = \frac{1-b/a}{\sqrt{|1-(b/a)^2|}}\sqrt{\frac{1-c/d}{1+c/d}}\,. \tag{19}$$

For the partially filled subband $m' = 5$ (at filling $\nu = 5.5$), the third argument of (2) is zero so that the real part of the dielectric function is logarithmic divergent around $\hbar\omega = 4\omega_c$ as can be seen from (18). In addition the real part of the dielectric function has four subsingularities for frequencies slightly larger than $4\omega_c$ and four subsingularities for frequencies slightly smaller than $4\omega_c$. The real part of the dielectric function is shown in Fig. (3) around $\omega = 4\omega_c$.

In a far infrared absorption or a Raman scattering experiment where the polarizations of the incoming and scattered photon are parallel, the charge-density excitations are measured. That is the scattering cross section which is proportional to $-\mathrm{Im}[1/\epsilon(q,\omega)]$. This function has peaks when $\mathrm{Re}[\epsilon(q,\omega)] = 0$. For an unmodulated system, the only peaks in the scattering

cross section are those due to the magneto-plasmon excitation. For a modulated system, certain spectral weight is shifted back to the energy corresponding to the electron-hole pair excitation. The peaks due to electron-hole pair excitation are rather sharp but finite even when disorders are negligible. Furthermore, we found that the cross section has an addition sharp peak at the $\omega = n\omega_c$ due to a logarithmic singularity in the real part of the dielectric function. This is depicted in Fig. (4).

In conclusion we have shown that a partial filling of the last Landau band may be detected in optical spectroscopy either in the spin-density excitation spectra or in the charge-density excitation spectra. The spectra will provide information about the modulation potential and the magnitude of the partial filling.

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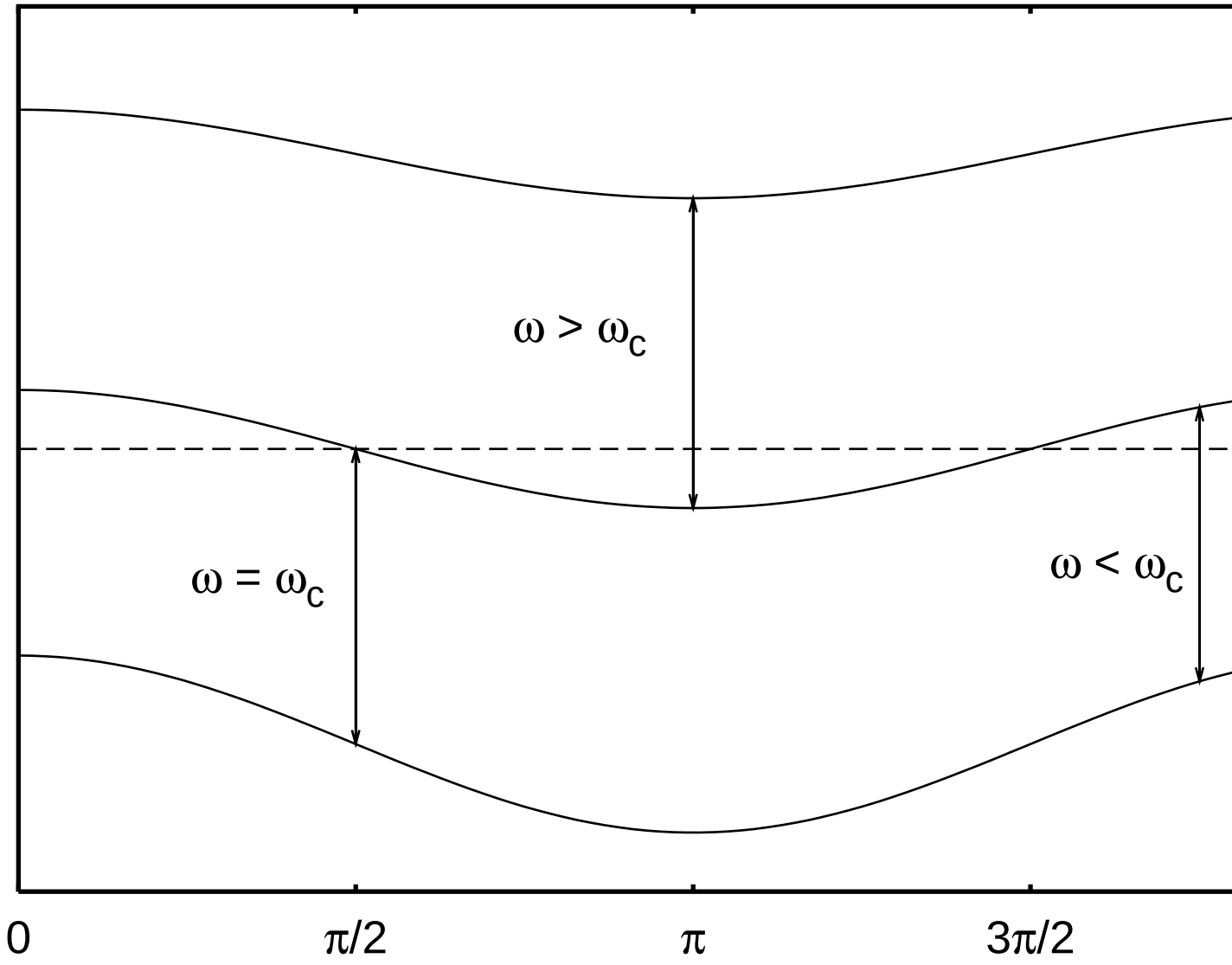
FIGURES

FIG. 1. Single-particle energies (lines) around the Fermi energy (broken line) as a function of the center coordinate, Kx_0, when the last subband is half-filled. Electron-hole pair excitations with energies $\omega < \omega_c$, $\omega = \omega_c$ and $\omega > \omega_c$ are shown.

FIG. 2. Imaginary part of the dielectric function,Im$[\epsilon(q_x, \omega)]$ as a function of ω/ω_c for $\nu = 5.5$, V_0=1 meV.

FIG. 3. Real part of the dielectric function,Re$[\epsilon(q_x, \omega)]$ as a function of ω/ω_c for $\nu = 5.5$, V_0=1 meV.

FIG. 4. $-$Im$[1/\epsilon(q_x, \omega)]$ as a function of ω/ω_c for $\nu = 5.5$, V_0=1 meV.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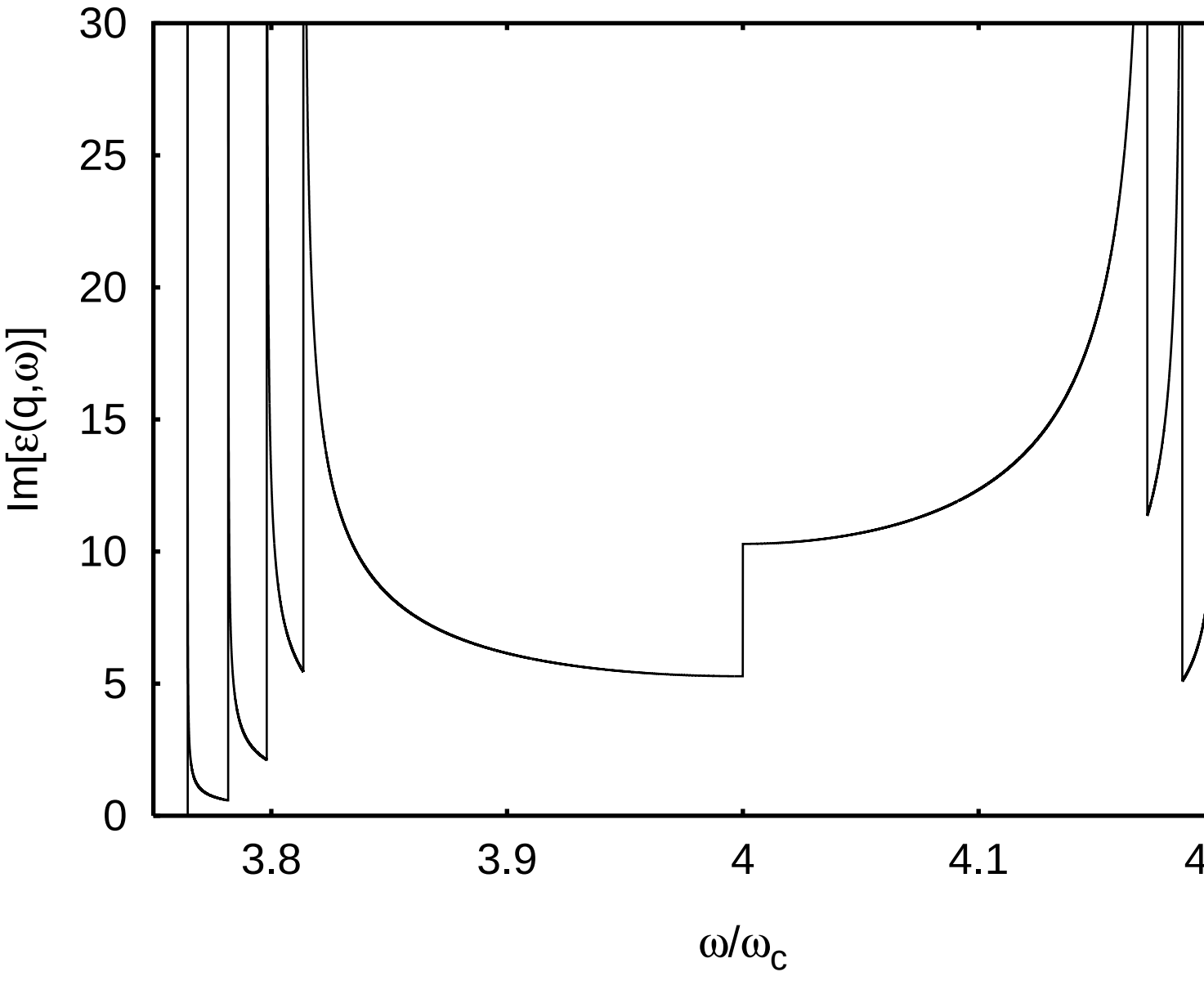
ω = ωc
ω < ωc
0
π/2
π
3π/2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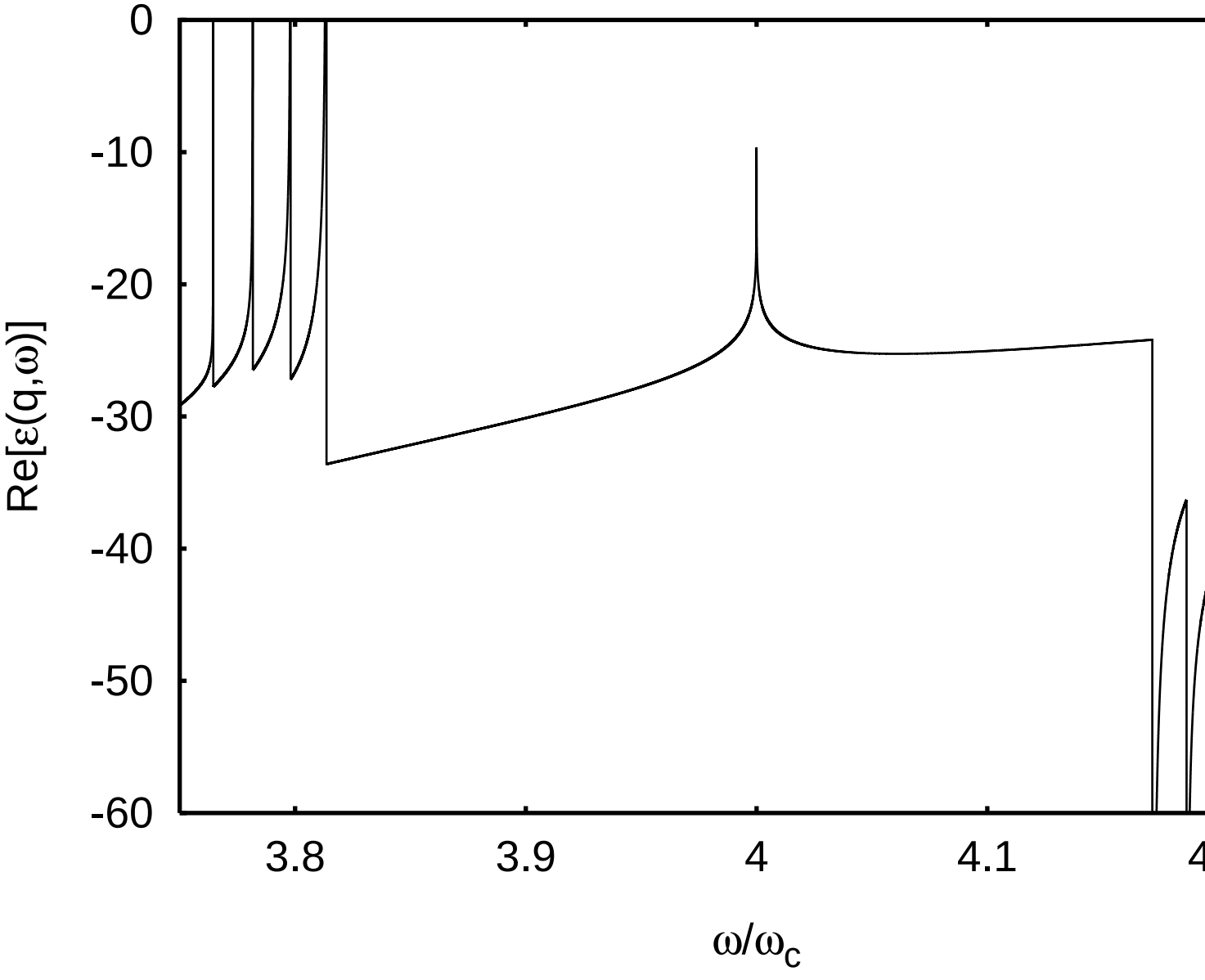
3.9
4
4.1
4
ω/ω_c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-40
-50
-60
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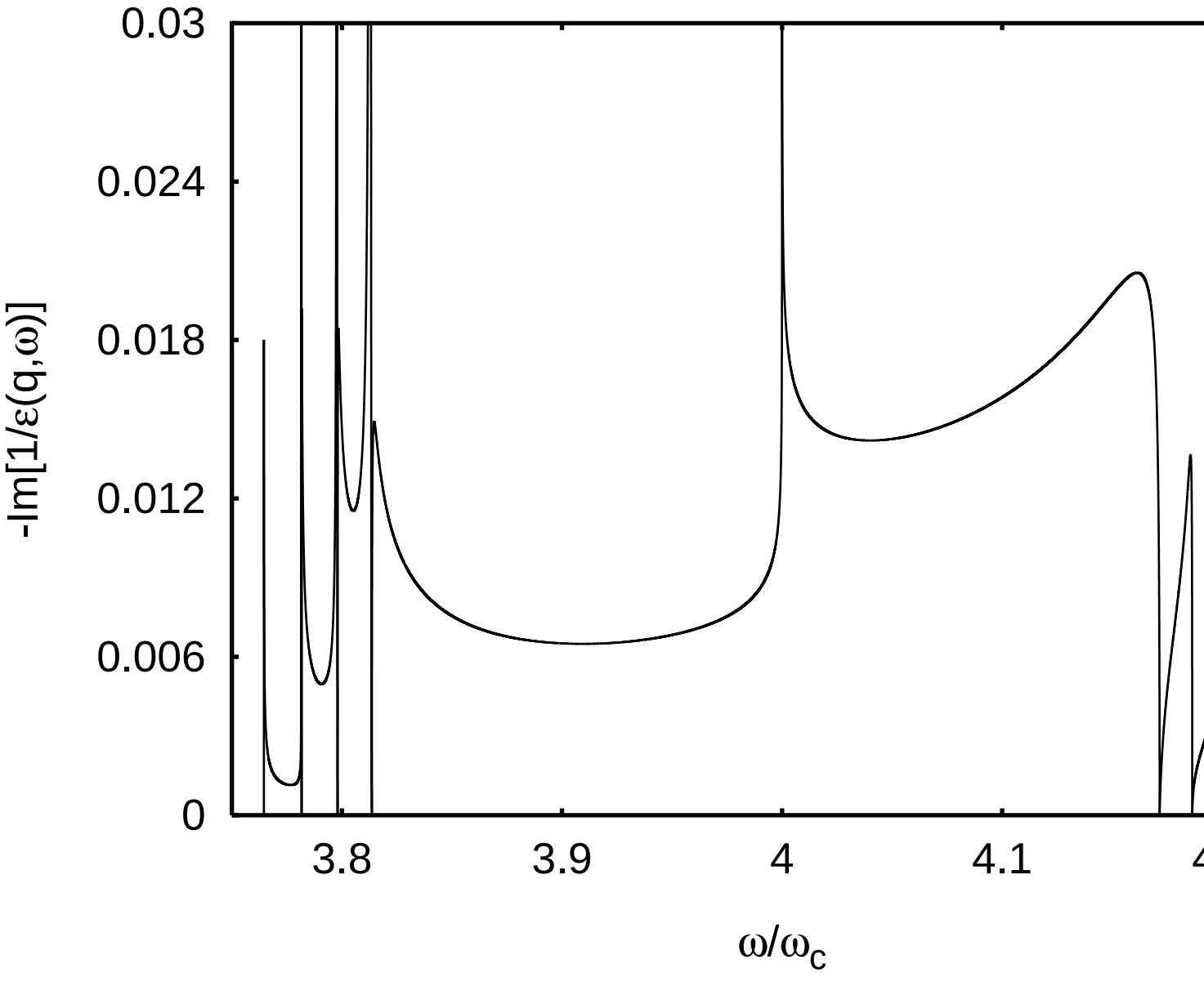
3.9
4
4.1
ω/ω_c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0.006
0
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3.8
3.9
4
4.1
ω/ω_c